Cooperation among mobile individuals with payoff expectations in the spatial prisoner's dilemma game

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We propose a model to address the problem how the evolution of cooperation in a social system depends on the spatial motion and the payoff expectation. In the model, if the actual payoff of an individual is smaller than its payoff expectation, the individual will either move to a new site or simply reverse its current strategy. It turns out that migration of dissatisfied individuals with relatively low expectation level leads to the aggregation of cooperators and promotion of cooperation. Moreover, under appropriate parameters migration leads to some interesting spatiotemporal patterns which seems not to have been reported in previously studied spatial games. Furthermore, it also found that a population with constant expectation can better favor cooperative behavior than a population with adaptive aspiration.

1. Introduction

Cooperative behavior is ubiquitous in biological and social systems. Since natural selection apparently favors those who can receive the benefits of mutualism without providing anything in return, the emergence and maintenance of cooperation in population of selfish individuals become a fascinating topic that has stimulated extensive studies in a variety of disciplines ranging from biological and social sciences to statistical physics [1–6]. Game theory [7,8] provides a uniform frame to study the evolution of cooperation, and one of the most commonly used as a paradigm for investigating this issue is prisoner’s dilemma game (PDG). In this game two players can choose either cooperation or defection. They both obtain payoff $R$ for mutual cooperation, but a lower payoff $P$ for mutual defection. A defector exploiting a cooperator gets the highest payoff $T$, while an exploited cooperator receives the lowest payoff $S$. We have $T > R > P > S$. It is easy to see that defection is the better choice regardless of the opponent’s selection. Therefore, in the absence of other assumptions, defection strategy will prevail in any mixed population.

Mechanisms supporting cooperation among competitive individuals have been broadly investigated [9]. Notably, since the pioneer work of Nowak and May [10], spatial structure, which allows cooperators to form clusters to resist invasion of defectors, has been well recognized as one of the key mechanisms for the emergence and persistence of cooperation. In the original Nowak–May spatial PDG model, individuals are placed on a two-dimensional array and interacting with their neighbors. Since then much effort has been expended on studying the evolution of cooperation in more complex topologies such as small-world [11,12] and scale-free networks [13,14]. In recent years the co-evolution of network structure and game strategy has also been the subject of intense investigations [15–18].

Spatial movement is one of the main ingredients of evolutionary dynamics and can also be interpreted as a co-evolutionary process [19,20]. However, for a long time, little attention has been paid to the role of migration. This is partially due to a comparatively prevalent viewpoint that mobility is a limited factor for the evolution of cooperation, because the moving of players provides an opportunity for defectors to invade and destroy the cooperator cluster. It is only in the very recent past that the role of mobility in evolutionary games has began to gain more and more attention [21–28].
Vainstein and Arenzon [21,22] extended the original Nowak–May PDG model by considering a regular lattice where some sites are empty, which allows diffusion of individuals. They found that, on the one hand, mobility may indeed promote cooperation, since it increases the ability of cooperator clusters to invade and overtake isolated defectors; on the other hand, mobility may also allow defectors to escape retaliation from a former partner and lead to stronger mixing in a population due to increasing interaction ranges of individuals, both of which are known to damp the evolutionary success of cooperators. Their study indicated that the effects of mobility on the cooperation are complex.

In the models proposed by Vainstein and Arenzon [21,22], the moving of individuals was considered to be Brownian random walk-like. Recently, some different co-evolutionary rules have been investigated by taking into account personal preferences in the movement of individuals. Helbing and Yu [24] introduced the idea of success-driven migration, which was based on the hypothesis that the individuals tend to move to the neighbor area with the highest expected payoff. The main finding is that success-driven migration can lead to cooperation in population of selfish and unrelated individuals even under noisy conditions.

Because the success-driven migration requires the non-local information and the computational capabilities of individuals for prediction, which is a strong requirement for individuals, Jiang et al. [26] proposed an alternative migration strategy that requires only local information obtainable through game interactions. In their model, an individual moves to an empty site with a probability proportional to the number of defectors in its neighborhood. After that, the individual updates its strategy by comparing its payoff with its neighbors’ payoffs, or reset its strategy with a very small probability. The simulation results indicate that there exists an optimal degree of migration, which can lead to the highest level of cooperation. It was also found that adaptive migration can induce an outbreak of cooperation from an environment dominated by defectors.

Recently, there has been an increasing interest in studying the effect of aspiration on the evolution of cooperation [29–32]. A new mechanism called aspiration-induced migration has also been proposed [27,28], in which an individual will migrate to a new site provided that its payoff is below some aspiration level, and then change its strategy following the one with relatively higher payoff in its neighborhood. As can be seen in many cases [26,29–31,33] that intermediacy in quantities can best favor cooperative behavior, which has been observed first in coherence resonance in a spatial prisoner’s dilemma game [34], the simulation results of aspiration-induced migration models [27,28] show that the highest cooperation level can be achieved when the aspiration level and interaction radius are moderate.

Thus, the recent studies [22,24,26–28] show that in certain conditions the migration of players may improve the formation of cooperator cluster and thus promote cooperation. In all of these models, a key mechanism called “imitation”, namely, the strategy of the best performing individual will be copied by its neighbors, plays an important role in the promotion of cooperation. The imitation rule enables cooperator cluster to expand by invading and overtaking isolated defectors. In this paper we introduce an alternative mechanism for promoting and sustaining cooperation in the population consisted of movable individuals. In our model all the individuals have a common payoff expectation. When an individual dissatisfies its payoff gained from its neighbors, it just moves to a new site or simply reverses its current strategy. Our simulation results show that, even without the imitation, at relative low values of payoff expectation the spatial migration can drive the cooperators to aggregate and form stable spatial patterns from self-organization of individuals.

2. Model

In our model, we assume that each individual has a payoff expectation $E$, a concept similar but different to the parameter, payoff aspiration level $P_{ia}$, introduced in the previous models [28–30]. The aspiration level $P_{ia}$ for an individual $i$ is defined as $P_{ia} = k_i A$, where $k_i$ is the number of neighbors of $i$ and $A$ is a control parameter. Therefore, the aspiration level of an individual in these previous studies is adaptive, depending on the number of neighbors. While in our model, we define payoff expectation $E$ as a constant number. We will show that different definitions of aspiration level $P_{ia}$ and expectation level $E$ lead to very different results in our present simulation frame.

In the simulation we arrange $N$ individuals of two subpopulation, cooperator and defector, on a square lattice of $L \times L$ sites with periodic boundary condition. With $N < L \times L$, each site is either empty or occupied by one individual. Individuals are updated asynchronously in a random sequential order, and as usual, one time step is defined such that each individual has chosen once on average. When an individual is selected randomly, it performs simultaneous interactions with the $m = 8$ near neighbors (Moore neighbor). Then the focal individual compares the cumulated payoff gained from its neighbors to its payoff expectation $E$ which is the same for all individuals. Considering that in real life when an individual dissatisfies its current situation, it can either migrate to a new environment or stay at current location but change its behaviors, we assume that if the actual payoff is less than the expected payoff, the focus individual has two choices: either to move to a randomly selected empty site within its eight neighboring sites with a probability $m$ (We also assume that if an individual chooses migration but all its eight neighboring sites have been occupied, the individual just do nothing in this iteration), or simply to reverse its current strategy with a probability $1 - m$. Here, $m$ represents the mobility rate of the population, and we investigate how the payoff expectation $E$ and $m$ influence cooperative behavior.

In the simulation the parameters are set as follows unless otherwise specified: The value of payoffs for PDG are $T = 1.3, R = 1, S = 0$ and $P = 0.1$ following the previous work [24]. The fraction of occupied sites is $\eta = 0.5$, and initially 50% of the individuals are cooperators and 50% are defectors. The average fractions defined in the section of Results,
including the fraction of cooperators $\rho_C$, the fraction of satisfied cooperators (the cooperators gain payoffs not less than their expectation $E$) $\rho_{SC}$, the fraction of defectors $\rho_D$, and the fraction of satisfied defectors $\rho_{SD}$, are obtained by averaging over 1000 time steps after a transient period of 25000 time steps for one realization and each data point is averaged over 100 independent realizations, and the simulations are performed on square lattices with $50 \times 50$ sites. The typical snapshots of spatial patterns of cooperators and defectors are obtained after $t = 1 \times 10^5$ time steps of simulations performed on $100 \times 100$ lattices, and the blue, red, yellow, and green colors used in the snapshots represent a cooperator, a defector, a cooperator who turned into a defector, and a defector who became a cooperator in the last iteration, respectively.

3. Results

3.1. Model with $3T + P = 4$

Fig. 1 shows the dependence of cooperation fraction $\rho_C$ on the mobility $m$ for different payoff expectations $E$. In the case of $m = 0$, when the individuals dissipate their cumulated payoffs they can only reverse their strategies but will not migrate. It turns out that the fraction of cooperators $\rho_C$ is about 0.5 in this condition. At another extreme $m = 1$ the individuals just try to move to a nearby empty site but do not change their strategies when they are in an unfavorable circumstance. In this case the cooperation level will always remain the same as in the initial state, $\rho_C = 0.5$. Between these two extremes, $0 < m < 1$, where an unsatisfied individual can either move to a nearby empty site with a probability $1 - m$, cooperation is found to be obviously enhanced in a broad range of the mobility parameter $m$ when $E = 2$ and $E = 3$. It is also interesting for $E = 4$ that the cooperation is inhibited at relative low mobilities, but is enhanced when $0.7 \leq m < 1$. To investigate why the cooperation can be enhanced in populations with relatively low payoff expectation $E$ when $0 < m < 1$, and what mechanism leads to the different results for $E = 4$ from other values of $E$, the snapshots of typical distributions of cooperators and defectors for different $m$ and $E$ on $100 \times 100$ square lattices are shown in Fig. 2.

The first column of Fig. 2 shows the never-move case ($m = 0$), in which many green or yellow spots illustrate that most of individuals keep changing strategies even after a long transient period of $10^5$ time steps. This scenario can be explained as follows: With the random initial configuration most players cannot be satisfied for $E \geq 1$ and they try to change strategies. However, since they do not move, changing strategies will result in another unsatisfied environment. Consequently, the players have to keep switching strategies between cooperation and defection. As a result, the fraction of cooperators $\rho_C$ is around 0.5 on average at $m = 0$. The last column of Fig. 2 is for $m = 1$, where the individuals do not change their strategies and rely solely on migration for trying to get a satisfactory reward. At $m = 1$, the cooperation level always keeps at 0.5. We find that most of cooperators move together and form stable clusters with boundaries covered by a few defectors, and the rest of defectors cannot find satisfactory position and have to keep wandering in the open space.

In the case of $0 < m < 1$, Fig. 2 shows that, for $E = 2$ and $E = 3$ migration and strategy-changing drive system to a "frozen state", in which players are no longer moving or strategy-changing because each individual can find a suitable position and an appropriate strategy to satisfy its expectation. Similar to the $m = 1$ case, cooperators are able to form steady clusters since they can receive high reward by mutual cooperation, whereas defector clusters are unstable because the payoffs obtained from mutual defection are low. However, the cooperation level at $0 < m < 1$ is quite different from the $m = 1$ case. A heuristic explanation is as follows: At $0 < m < 1$, an unsatisfied individual will have an uncertain strategy and cannot settle down. Considering that a stable cluster will mainly consist of cooperative players, strategy-changing will provide relatively more chances for a defector to change to a cooperator than for a cooperator to change to a defector. As a result, a high cooperation will be obtained in the system, as shown in Fig. 1. Additionally, For $E = 2$, a cooperator will be satisfied and settle down when there are at least two of its neighbors are cooperators, while for $E = 3$, a stabilized cooperator requires at least three neighbor cooperators. Thus the size of the cooperative clusters formed in the population with $E = 3$ are larger than those for $E = 2$, resulting in a higher cooperation level for $E = 3$.

We also find that (data not shown), for $E = 1$, since individuals can be satisfied by a single cooperative neighbor, the system will become stabilized quickly with very small defector clusters and cannot promote the growth of cooperative cluster significantly. While for $E \geq 5$, high expectation makes it difficult for individuals to find enough cooperative neighbors to meet their demands. Consequently, the individuals have to change their strategies or locations very frequently, which makes the system unstable and hardly to form cooperative clusters, resulting in about the same cooperation level as the initial state.

The spatial patterns for $E = 4$ shown in Fig. 2 are significantly different from those for $E = 2$ and $E = 3$. There is no cooperative cluster formed at low mobilities in the popula-
tions with $E = 4$. However, when the value of $m$ increases, regular configurations which look like pipelines appear in the system. The inner layer of such a pipeline is formed by the aggregation of cooperators, and the outer layer consists of defectors. To investigate how this type of configuration is formed, the snapshots shown in Fig. 3(a–d) present the temporal evolution of spatial patterns for $E = 4$ and $m = 0.9$.

It is found that at the beginning a few cross-shaped clusters formed by 12 cooperators emerges in the system (Fig. 3(e)). In such a configuration, each cooperator has at least four cooperative neighbors, hence they all satisfy with their payoffs obtained from the game and no longer change their strategies or positions. Once a stable cross-shaped cluster is formed, it can be used as a "seed" from which a larger stable cooperator cluster can be grown. As shown in Fig. 3(f), based on such a minimum stable unit, five cooperators can be attached to one of the four corners of the cross-shape clusters, resulting in the elongation of the pipeline along one direction. Furthermore, the configuration shown in Fig. 3(f) provides adhesion points for four defectors (Fig. 3(g)). Adhesion of defectors forms the shell of the pipeline (Fig. 3(h)), in which each defector gains satisfactory payoff $3T + P = 4$ from three cooperator neighbors and one defector neighbors. Once the shell of defectors is formed, the cooperators can no longer join the pipeline in the radial direction. However, in the axial direction of pipeline (northeastward or southwestward in Fig. 3(h)), more individuals will continue to join the cluster through the process described above, which leads to the elongation of pipeline until the ends of pipeline meet the other pipeline or there is no enough individuals to adhere to the pipeline.

The results presented above show that, even without imitation mechanism, migration can drive dissatisfied individuals to find suitable environment, allowing the cooperators and defectors to form some stable configurations and settle down. From Fig. 2 one can see that differ-

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**Fig. 2.** Typical snapshots of spatial patterns of cooperators and defectors for different payoff expectations $E$ and different mobilities $m$.

**Fig. 3.** (a–d) Temporal evolution of spatial pattern for payoff expectation $E = 4$ at different times. (e–f) Schematic diagrams of how a pipeline-like configuration is formed by cooperators and defectors.
ent expectations $E$ lead to different types of stable configurations. To provide a further understanding of the results shown in Fig. 1 and Fig. 2, we arrange a varying number of individuals in some small square lattices, and investigate how many stable configurations can be formed by various permutations and combinations of cooperators, defectors, and empty sites. In Fig. 4(a), we count the total number of possible stable configurations $n_s$ that can be formed by individuals with different values of $E$ in $2 \times 2$, $3 \times 3$ and $4 \times 4$ square lattices. In Fig. 4(b) we measured the proportion of cooperators $f_C$ averaged over all these stable configurations.

The data shown in Fig. 4 explains qualitatively many aspects of our simulation results. It is found that, when $E < 4$, the individuals can form a large number of stable configurations (Fig. 4(a)). As a result, the individuals with $E < 4$ can easily find suitable position to settle down and the system will reach steady state in a relatively short period of time. From Fig. 4(b) one can see that, on average, cooperators contribute more than defectors in forming these stable configurations, i.e., $f_C > 0.5$, which explains why in the steady state the fraction of cooperators $f_C$ is larger than 0.5 for a population with a relatively small value of $E$. Moreover, since $f_C$ increases linearly with increasing expectation $E$ (Fig. 4(b)), it is understandable that a population with $E = 3$ will evolve to a higher cooperation level than that with $E = 2$, and a population with $E = 2$ will maintain a higher cooperation level than that with $E = 1$, as shown in Fig. 1.

As for $E = 4$, the situation is different. Fig. 4 indicates that there is only one stable configuration in a $4 \times 4$ lattice, i.e., $n_s = 1$ in Fig. 4(a). Forming such a minimum stable configuration requires concurrence action of twelve individuals (Fig. 3(e)). They should be cooperator synchronously in the right position and at the right time to form a cross-shaped stable cluster. Moreover, the elongation of pipeline structure also needs simultaneous adhesion of five cooperators (Fig. 3(f)). This leads to two effects. One effect is that it is more difficult for individuals with $E = 4$ to settle down in a satisfied environment than those with smaller value of $E$. As a result, it will take longer time for system to reach steady state. Another effect is that, at a low mobility $m$ and thus a high strategy change rate $1 - m$, there is little chance for individuals to form the initial cooperative pipeline-like individual clusters, resulting in a low cooperation level; while a relatively high mobility can cause the formation of cooperative seeds and enhance the cooperation in the system.

As shown in Fig. 1, in the low value of $m$ for $E = 4$ and also in a large range of $m$ for $E = 5$, the cooperation level is lower than 0.5, showing an inhibition effect on the evolution of cooperation. In order to understand this result, we count the fraction of cooperators $\rho_C$, the fraction of satisfied cooperators (the cooperators gain payoffs not less than their expectation $E$) $\rho_{SC}$, the fraction of defectors $\rho_D$, and the fraction of satisfied defectors $\rho_{SD}$ with different $E$ for $m = 0.3$ and $m = 0.9$.

As shown in Fig. 5(a) and (b), no matter what the value of $m$ is, when $E < 3$, $\rho_C = \rho_{SC}$ and $\rho_D = \rho_{SD}$, indicating that all the individuals are satisfied with their payoffs and settle down. While in the case of $E > 5$, $\rho_C = \rho_{SC} \approx 0.5$ and $\rho_D = \rho_{SD} \approx 0$, which means that none of the individuals can find enough cooperators to satisfy their demands. All the individuals, either cooperators or defectors, can only keep moving or changing their strategies. Since neither cooperators nor defectors have a competitive advantage, the cooperation fraction will keep at $\rho_C \approx 0.5$, the same as the initial state (Fig. 1).

As for $E = 5$, most of the time the individuals are in the dissatisfied states, but occasionally the defectors would be surrounded by enough cooperators and become satisfied instantaneously (Fig. 5(c) and (d)), which entails the defectors a slight advantage ($\rho_{SD} > \rho_{SC} \approx 0$). As a result, the cooperation level is slightly lower than 0.5 for $E = 5$ on average. The results for $E = 4$ are largely depended on the mobility $m$. In a low mobility $m$, the pipeline-like structure cannot be formed, and the individuals keep changing their states. Therefore, similar as the case of $E = 5$, the defector has a slight advantage comparing to cooperator (Fig. 5(c)), causing the inhibition of the cooperation and thus $\rho_C < 0.5$. While in the high mobility, pipeline-like structures enhance the cooperation as in the case of $E = 2$ or $E = 3$ (Fig. 5(d)).

3.2. Model with $3T + P < 4$

From the analysis above we can see that the formation of pipeline-like shape for $E = 4$ is due to the special structure of the payoffs matrix for PDG, which satisfies $3T + P = 4$ and enables the defectors to aggregate to the outer layer of the cooperator cluster, forming the shell of the pipeline. Once the shell is formed, the cooperator can only join in the pipeline-like cluster in axial direction. Therefore, if the payoffs matrix does not satisfy the conditions, the pipeline-like pattern will be destroyed. Fig. 6 shows the results for $3T + P < 4$. In this case the defector shells are no longer stable. Indeed, one can see that in a very large range of value of $m$ the cooperator cluster can become larger without the inhibition of the defector shell, which enables the system to evolve to a very high cooperation level for $E = 4$. Thus the self-organized pattern formation of the cooperators and defectors depends strongly on the payoff expectation levels and the payoff matrix for PDG.
3.3. Model with adaptive aspiration

So far we assume that each individual has a constant payoff expectation $E$, which is different from the payoff aspiration $P_{ia} = k_iA$ adopted in the previous study [27]. It is interesting to investigate how the adaptive aspiration affects the evolution of cooperation in our model. For this purpose, we simulate the evolution of cooperation in our model consisting of individuals with aspiration $P_{ia}$. Fig. 7(a) shows the cooperation fraction in the steady state for different values of $A$ and $m$. We find that in our model, without imitation rule, aspiration-induced migration cannot promote cooperation. The typical snapshots of spatial patterns shown in Fig. 7(b)–(d) indicate that the cooperators cannot aggregate and form relatively large clusters, resulting in low cooperation levels.

It is understandable that for high aspiration level the cooperators cannot aggregate to clusters due to the frequent movement and strategy change of dissatisfied individuals. But why in relatively low aspiration level, the cooperators still cannot aggregate as they do in the case of constant expectation level? The answer lies in the fact that the aspiration level $P_{ia}$ of an individual is adaptive, depending on the number of its neighbors $k_i$. When $k_i$ decreases, the individuals will lower their aspiration level accordingly. As a result, the individual may settle down even if the number of neighbors are small, which makes it difficult for cooperators to form large clusters. On the contrary, an individual with constant payoff expectation requires not the average payoff, but the total payoff it can obtain from its neighbors to fulfill its demand. Consequently, in such a system the players tend to migrate to...
the places with lots of individuals, enabling the cooperators to form relatively large cluster, which promotes and maintains the cooperation in movable populations. However, it should be noted that if the density of individuals $\eta$ is lower than 0.5, then at a relatively high constant expectation level $E$ the comparison of the impacts of expectation and adaptive aspiration levels may be different. For example, when we set $\eta = 0.3$ and $E = 4$, we found that (data not shown) it is hard to grow the pipeline-like cooperator clusters because the cooperative players cannot collect enough payoffs due to less links. Thus the cooperation cannot be enhanced in this situation. While the density of players has little influence on the adaptive aspiration case, because the aspiration is normalized and thus independent from the actual links of a player. Therefore, how the density of players affects the cooperation level in the situation of constant expectation and adaptive aspiration needs to be further investigated.

4. Conclusions

In summary, we have explored how the spatial payment and payoff expectation of individuals influence the evolution of cooperation. In our model some simple game strategies are considered for individuals. In detail, when an individual dissatisfies its payoffs it will either migrate to a new position nearby, or just reverse its current strategy. Therefore, without imitation rule, the individuals require less information and computation than those in previous studies [24,26–28]. We find that the population can also favor cooperative behaviors when the expectation levels of the individuals are relatively low.

The mechanism that enhances the cooperation in our model is that, with the expectation $E < 4$, there exist lots of possible stable configurations in which each individual satisfies its payoffs and on average there are more cooperators than defectors in these stable configurations. As a result, when the system reaches a steady state and most of the individuals have found their positions in one of such stable clusters, the number of cooperators is larger than that of defectors on average. As for $E = 4$, the possible number of stable configuration is much smaller than that for $E < 4$. However, given enough evolution time and a large migration rate, the individuals can also aggregate and form regular spatial patterns from self-organization, which maintains a relatively high cooperation level in the system.

Acknowledgments

This work is supported by the National Science Foundation of China under Grant No. 30970970 and Ministry of Education Grants 108075 and 20090121110028.

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